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MIXED METHOD
IN MATERIAL MODELING OF QUASIBRITTLE MATERIAL

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ABSTRACT: A material model coupling together the microplane approach with a damage transformation of the strain tensor field in cracked continuum will be presented. The issue has been divided in two parts. In the present report, the local material behaviour is analysed, in the second one the nonlocal concepts will be discussed with the application of the material model to the analysis of structures.

Starting with a review of the standard microplane model, a triaxial numerical test has been carried out to understand the reasons for the poor performance of this model in postpeak tensile behaviour. When the microplane model parameters are chosen so as to fit the experimental data in triaxial compression tests, the resulting tensile behaviour is unsatisfactory. The same happens the other way round. No successfull compromise could be found. For this reason, a solution was investigated where a separate modeling of the tensile failure and the crack opening is used together with the basic features of the microplane model. Theory and verification of the new model is presented together with a detailed comparison between the results of the two models.

INTRODUCTION

Numerical simulation of nonlinear fracture behaviour can be performed in many different ways. We witness today a strong development of micromodels where the fracture mechanism is decomposed into its basic components. With surging computer performance these approaches seem very promising and probably will set the standards for tomorrow's research.

Also the neural networks will be exploited giving us a completely new approach to the material modeling. Neural computing should hopefully overcome the main problem of all the existing models todate, where material behaviour must be predefined in close form expression, whether on global level as in the macromodels or at subcomponent level as in the microplane model and in all the micromodels in general.

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At the moment the drawback of most of these approaches is that they cannot be used in standard discrete solution techniques. Here instead, we are only interested in the material models that can be used to model constitutive behaviour in finite element analysis without the need of special technique like remeshing or discrete interface modeling. A smeared contitutive model for quasibrittle materials that can be used in the finite element analysis is today very much needed.

As a matter of fact, there are plenty of models that have been claimed to be successfull in achieving the task or at least part of it. If anything, the microplane model has proved to have some advantages with respect to both smeared crack models and plasticity based macromodels. This advantage is due to the fact that microplane is a model that comes from the integration of simple behaviours on planes of various orientations. This means that the material model in macroplane is treated as a structural element itself, with boundary conditions, kinematic and equilibrium equations and constitutive relations. This approach can improve the problems found using the smeared crack approach *Petrangeli and Ožbolt 1992* still retaining simplicity in the input needed for the basic microplanes stress-strain relations.

The idea of integrating simpler behaviour on planes to obtain a three dimensional model has now been around for more than half a century. The model has been used with both kinematic and static constraint.

Static constraint was implemented at the beginning when the model was used for steel and general plastic behaviour. Starting in 1938 with G.I.Taylor and following with Batdorf and Budinaski in 1949, under the name of "slip theory of plasticity" a very similar concept to today's microplane was developed *Taylor 1938, Batdorf and Budinaski 1949*. This model was integrating shear behaviour on different planes, this way the mechanism of crystal plane dislocation could be modelled with accuracy and simplicity at the same time.

In the later version by *Bažant and Prat (1988) and Ožbolt and Bažant (1991)*, the model relies on the kinematic constraint. When softening behaviour is involved, this approach greatly simplify the computational task providing, together with a secant or constant stiffness iteration method, a simple and stable solution strategy.

On the other hand, the kinematic constraint fails to describe realistically the strain field when strong localization takes place due to cracking. In this case, the continuous projection of large tensile strain on all the different orientations (i.e. the microplanes) is inconsistent and produces unrealistic lateral strains (see Fig.1). This problem can neither be solved by adjusting the microplanes parameters for the constitutive equations now implemented in the model, nor with a new different set of them.

This last possibility has been pursued in the last few years with unsatisfactory results by different authors, yet a systematic literature on this subject does not exist *(see Carol and Prat 1991; Carol, Bažant and Prat 1992)*.

KINEMATIC APPROACH - STANDARD MICROPLANE

Let's now review the microplane model based on the kinematic approach before discussing the problems given by this approach when fracture process are to be modelled. Using this approach, strain on microplanes are found by standard transformations of the total strain tensor, the total strain on a microplane whose direction cosines are n_i is:

$$\epsilon_j^n = \epsilon_{jk}\, n_k \tag{1}$$

Total strain on planes can then be decomposed in its normal and tangent component according to the relations:

$$\epsilon_T = \epsilon^n - \epsilon_N \tag{2}$$

$$\epsilon_N = n_j\, \epsilon_j^n = n_j\, n_k\, \epsilon_{jk} \tag{3}$$

$$\epsilon_{N_i} = n_i\, n_j\, n_k\, \epsilon_{jk} \tag{4}$$

$$\epsilon_{T_i} = (\delta_{ij} - n_i n_j) n_k \epsilon_{jk} \tag{5}$$

$$\epsilon_T = \sqrt{\epsilon_{T_i} \epsilon_{T_i}} \tag{6}$$

The normal component can be further split into a volumetric and a deviatoric part as suggested by *Bažant and Prat (1988)*. This was needed for different reasons that we will not discuss here but can be found in the references.

$$\epsilon_V = (\epsilon_N + \epsilon_K + \epsilon_M)/3 \tag{7}$$

$$\epsilon_D = \epsilon_N - \epsilon_V \tag{8}$$

From strains, stresse are found using simple constitutive relations. For each microplane we can write:

$$\sigma_D = C_D(\epsilon_D)\,\epsilon_D \tag{9}$$

$$\sigma_T = C_T(\epsilon_T, \epsilon_V)\,\epsilon_T \tag{10}$$

The following expressions are used for the microplane secant moduli C_D and C_T:

for $\epsilon_D \geq 0$:

$$C_D = C_D^0\, e^{-|\epsilon_D/a_1|^{p_1}} \tag{11}$$

for $\epsilon_D < 0$:

$$C_D = C_D^0\, e^{-|\epsilon_D/a_2|^{p_2}} \tag{12}$$

$$C_T = C_T^0\, e^{-|\epsilon_T/a_5|^{p_3}} \tag{13}$$

where a_1 and a_2 are constants while a_5 depends also on the volumetric strain via the expression $a_5 = a_3 - e_4\epsilon_V$. The previous expression is needed to model material behaviour at high confining pressure, a_3 and a_4 are constants.

The volumetric stress strain relations of scalar type, have the following similar expressions:

$$\sigma_V = C_V(\epsilon_V)\,\epsilon_V \tag{14}$$

for $\epsilon_V \geq 0$:

$$C_V = C_V^0 e^{-|\epsilon_D/a_1|^{p_1}} \tag{15}$$

for $\epsilon_V < 0$:

$$C_V = C_V^0 \left[\left(1 + \left|\frac{\epsilon_V}{a}\right|\right)^{-p} + \left|\frac{\epsilon_V}{b}\right|^q \right] \tag{16}$$

where a, b, p and q are also constants. The total stress vector is then found using an energy equivalence between work done on macro level, by macro strains, and on micro level, by microplane strain components.

$$\frac{4\pi}{3}\sigma_{ij}\delta\epsilon_{ij} = \int_S (\sigma_N \delta\epsilon_N + \sigma_T \delta\epsilon_T)\, dS \tag{17}$$

A fundamental requirement for this model is to keep the stress strain relations on microplanes as simple as possible. Complexity of the material behaviour should be achieved by integration over different directions and not with the local constitutive laws. Otherwise the model ends up having the same problems of the macroscopic ones, where the correlation between different orientations needs to be predefined in the material tensor. As a matter of fact, microplane model as presented in the work of *Bažant and Prat (1988)*, succeeded in reproducing different fundamental features of concrete behaviour using very simple stress strain laws on microplanes.

Few things need to be said on the kinematic approach, i.e. defining microplane strains with Eq.(1). This aproach has many advantages, they basically fall into one of the following categories:

- solving microplane strains from total ones fits with the standard displacement solution used in most finite element codes and in the finite elements themselves. From the solved nodal displacement, using the standard finite element shape functions, deformations at Gauss points are found, and from these, with Eq.(1-8) the microplanes ones. The algorithm is extremely time efficient and stable even when the solution is in softening regime.

- by choosing a kinematic approach throughout the solution procedure, if the material model is consistent, problems of mesh dependency do not arise. If the mesh is fine enough, independently of its shape and orientation, the solution is unique. By choosing always the stiffest approach we avoid following different local paths when bifurcation points arise due to softening.

At the material level the kinematic approach is basically equivalent to a parallel assumption of the microplane's behaviour, which means that softening in one plane does not entrain unloading on any of the other ones. This means that the softening on global stress-strain relation can be achieved only when all the microplanes involved are on the softening branch as well (not to be taken literally).

When it comes to modeling the tensile behaviour this approach is clearly unrealistic. Large tensile deformations which are due to crack opening, cannot be transformed continuosly into strains on different microplanes according to the kinematic constraint. When the crack develops, we still need to average the crack strain if we are to use a smeared model, but the compatibility equations needs to be modified.

How the standard microplane model tried to cope with it? To mitigate the overstiff response yielded by the kinematic constraint, very brittle stress strain relations on microplanes are used. When in a certain direction the material goes in softening regime, due to the kinematic constraint no unloading takes place on any of the other directions. All the microplanes keep loading, and the only way to release the stored energy is for all the strain components to go into softening. In the case of a static constraint instead, this would be achieved, due to the equilibrium, by the unloading taking place in all directions except for the softening one.

Though the static constraint is main responsible for the ill conditioned post peak behavior at large tensile strains, there are also some other situations where the results of the model are not fully consistent. We shall go through some of these in the next paragraph.

KINEMATIC APPROACH - NUMERICAL VERIFICATION

Numerical tests have been performed on a single square finite element with one integration point. Unit element dimensions have been chosen so that deformations and displacements coincide. The element uses the standard microplane version as described by *Bažant and Prat (1988)*. Material parameters have been set as follows: $a_1 = 0.00007$, $a_2 = 0.0017$, $a_3 = 0.0017$, $a_4 = 5.0$, $p_1 = 1.0$, $p_2 = p_3 = 1.4$, $a = 0.005$, $b = 0.0435$, $p = 0.75$, $q = 2.0$. Initial elastic Young's modulous was set to $30000 MPa$ and Poisson's ratio to 0.18.

First example is uniaxial tension. In *Fig.(1)* the tensile stress is plotted as a function